Combinatorics Between the World Wars

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General remarks

- Combinatorics was already a well-established mathematical discipline, new results published in respected journals (Math. Ann., J. Lond. Math. Soc., Amer. J. Math.)
- Key results were often discovered while investigating other topics (group theory, almost-periodic functions)
- Graph theory had a bad reputation as a science of trivial problems, new results often formulated in the language of matrices or topological structures; first textbook published in 1936
- Numerous results were motivated by practical problems (design of experiments, construction of electricity networks, enumeration of chemical compounds)

Main research topics in combinatorics

1. Latin squares and block designs, their relation to finite projective planes and algebraic structures, applications in the design of experiments
2. Beginnings of combinatorial set theory (Hall's marriage theorem and related results, Sperner's theorem)
3. Beginnings of Ramsey theory
4. Graph theory
5. Additional topics (Redfield–Pólya enumeration theory, Whitney's matroid theory, integer partitions)

Latin squares

A Latin square of order n is an array consisting of n rows and n columns. Each cell contains a number from the set $\{1, \ldots, n\}$ in such a way that each number occurs exactly once in each row and exactly once in each column.

There exist Latin squares of an arbitrary order n, but no efficient formula giving their total number is known.

- R. A. Fisher, F. Yates (1934): There exist 812 851 200 squares of order 6.
- B. D. McKay, I. M. Wanless (2005): Number of squares of order 11 (ca. $8 \cdot 10^{47}$)

Orthogonal Latin squares

A pair of Latin squares $\Rightarrow$ a square containing n^2 pairs of numbers from $\{1, \ldots, n\}$

1	2	3
2	3	1
3	1	2

1	2	3
3	1	2
2	3	1

1_1	2_2	3_3
2_3	3_1	1_2
3_2	1_3	2_1

If each ordered pair (i, j) occurs exactly once throughout the array, the two Latin squares are said to be orthogonal (also: a *Graeco-Latin square*).

Aα	Bγ	Cβ
Bβ	Cα	Aγ
Cγ	Aβ	Bα

Aα	Bγ	Cδ	Dβ
Bβ	Aδ	Dγ	Cα
Cγ	Dα	Aβ	Bδ
Dδ	Cβ	Bα	Aγ

36 officers problem

Leonhard Euler (1776, 1782):

36 officers problem: Is it possible to arrange six regiments consisting of six officers each of different ranks in a 6×6 square such that no row or column duplicates a rank or a regiment?

Denote regiments and ranks by letters:
a, b, c, d, e, f correspond to regiments, $\alpha, \beta, \gamma, \delta, \epsilon, \zeta$ correspond to ranks.

Each arrangement of the 36 officers corresponds to a 6×6 array consisting of pairs of letters; each ordered pair appears exactly once. Each Latin and each Greek letter is contained exactly once in each row and each column.

Hence, the problem is equivalent to the construction of a pair of orthogonal Latin squares of order 6.

Euler's conjecture

Leonhard Euler (1776, 1782):

- If n is odd or divisible by four, then there exist orthogonal Latin squares of order n.
- There are no orthogonal Latin squares of order 2.
- Conjecture: There are no orthogonal Latin squares of order 6. More generally, there are no orthogonal Latin squares of order $4k + 2$.

Gaston Tarry (1900): The conjecture holds for $n = 6$ (34 pages, analysis of 17 types of Latin squares)

Systems of orthogonal Latin squares

Do there exist systems of more than two Latin squares of order n, each two of which are orthogonal?
$N(n)$ = maximum number of mutually orthogonal Latin squares of order n

Euler's conjecture: $N(4k+2) = 1$

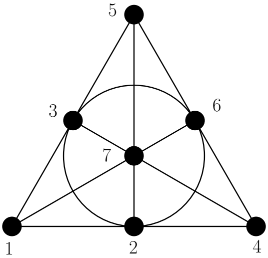
Observation: $N(n) \leq n - 1$

Harris F. MacNeish, *Euler Squares* (Ann. of Math., 1922):

- If n is a prime power, then $N(n) = n - 1$.
- Given k mutually orthogonal Latin squares of order m and k mutually orthogonal squares of order n, it is possible to construct k mutually orthogonal Latin squares of order mn. Thus, $N(nm) \geq \min(N(n), N(m))$.
- If $n = p_1^{r_1} \cdots p_k^{r_k}$, then $N(n) \geq \min(p_1^{r_1}, \ldots, p_k^{r_k}) - 1$. Does equality hold? (Generalization of Euler's conjecture)
- Wrong proof of Euler's conjecture

A finite projective plane is a pair of finite sets X (“points") and $P \subset \mathcal{P}(X)$ (“lines") with the following properties:

1. Every two lines $p_1, p_2 \in P$ intersect in a unique point ($|p_1 \cap p_2| = 1$).
2. Every two distinct points $x_1, x_2 \in X$ determine a unique line p ($x_1, x_2 \in p$).
3. There exist four points no three of which lie on a single line.



Finite projective planes (2)

The following statements hold for each finite projective plane:

- Every two lines have the same number of points.
- The number of lines equals the number of points.

If each line has $n+1$ points, we speak of a finite projective plane or order n.

- The smallest finite projective plane has order 2, and consists of 7 points and 7 lines.
- A plane of order n has n^2+n+1 points as well as lines.

For which numbers n does there exist a finite projective plane of order n?

Existence results (1)

Frank Yates, *Incomplete randomised blocks* (Annals of Eugenics), 1936: *If there exist $n-1$ orthogonal Latin squares or order n, then there exists a finite projective plane of order n.* (formulated in the language of block designs)

It is known that completely orthogonalized squares exist when the side is a prime number and also for sides 4, 8 and 9. It is also known that no such square of side 6 exists. Higher non-primes have not been investigated.

Existence results (2)

Raj Chandra Bose, *On the Application of the Properties of Galois Fields to the Problem of Construction of Hyper-Graeco-Latin Squares* (Sankhya: The Indian Journal of Statistics), 1938:

Professor Fisher, during his recent visit to India, in a Seminar held under the auspices of the Statistical Institute, made the surmise that it should be possible to construct a Hyper-Graeco-Latin square for every value of p, which is a prime or a power of a prime. The object of this paper to prove that this surmise is correct, by using the properties of Galois Fields.

Theorem

A finite projective plane of order n exists if and only if there exist $n-1$ orthogonal Latin squares or order n.

Explicit construction of orthogonal Latin squares using finite fields.

Algebraic connections

Let $n = p^k$, where p is a prime. Then there exists a finite field $GF(p^k)$ having n elements (Galois field):

$$GF(p^k) = \{g_1, \dots, g_n\}$$

For each choice $g \in GF(p^k) \setminus \{0\}$, the matrix with elements

$$a_{ij} = g \cdot g_i + g_j, \quad i, j \in \{1, 2, \dots, n\}$$

is a Latin square of order n.

Different choices of g lead to $n - 1$ distinct Latin squares, each two of which are orthogonal.

Later results

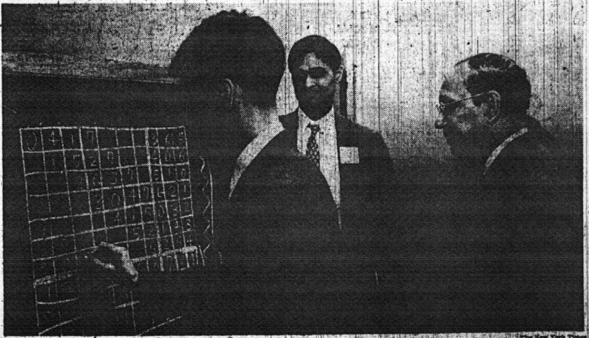
E. T. Parker, 1959: There exist 4 orthogonal squares of order 21 $\Rightarrow$ MacNeish's conjecture is wrong

R. C. Bose, S. S. Shrikhande, 1959: There exist orthogonal squares of order 22 $\Rightarrow$ Euler's conjecture is wrong

R. C. Bose, E. T. Parker, S. S. Shrikhande, 1960: For each $n \in \mathbb{N}$ except 2 and 6, there exist orthogonal Latin squares of order n.

The existence of finite projective planes whose order is not a prime power remains open. They do not exist for $n = 6$ (Tarry 1900), $n = 10$ (Lam 1991, computer search). The existence for $n = 12$ is an open problem.

Major Mathematical Conjecture Propounded 177 Years Ago Is Disproved



The New York Times

Discussing their solution to problem, from left: Dr. E. T. Parker, Prof. S. S. Shrikhande and Prof. R. C. Bose

By JOHN A. OSMUNDSEN

Another major mathematical problem—this one 177 years old—has been solved. Its solution was reported at the 557th meeting of the American Mathematical Association, which ended at the New Yorker Hotel yesterday.

It was the second such achievement to come out of the meeting, something attending mathematicians called "extremely rare." The solution to the first problem, known as Frobenius' conjecture, was reported by Prof. John G. Thompson, a 26-year-old mathematician from De-Paul University in Chicago. It dealt with so-called "group theory" and had puzzled mathematicians for more than fifty years. The second problem had resisted attempts at solution ever since Leonhard Euler (pronounced "oiler") stated it in a memoir in 1782. It became famous as Euler's conjecture. The three mathematicians who finally cracked the problem are now known among their colleagues as

Continued on Page 42, Column 1

Title page of New York Times, April 26, 1959

Latin squares and the design of experiments

The interest in Latin squares and related structures was fuelled by applications in the design of agricultural experiments.

The pioneer of the use of combinatorial and statistical methods in experiments was Ronald Aylmer Fisher (1890–1962). He spent the years 1919–1933 at the Rothamsted Experimental Station, where he worked on the design and analysis of field experiments.

Fisher's work was summarized in 3 highly influential books, all of which contain passages devoted to Latin squares: *Statistical Methods for Research Workers* (1925, 14 editions), *The Design of Experiments* (1935, 9 editions), *Statistical Tables for Biological Agricultural and Medical Research* (1938, 6 editions, 4 reprints).

The last book was written together with Frank Yates (1902–1994), who became assistant statistician at the Rothamsted Station in 1931, and took over Fisher's position in 1933.

An experiment with potatoes in Ely, 1932

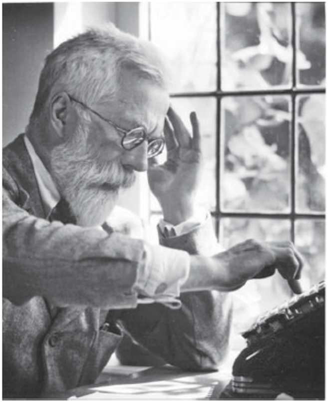
The goal of the agricultural experiments was to verify the effectivity of various fertilizers, herbicides, insecticides, etc.

An example from Fisher's book *The Design of Experiments*:

E	B	F	A	C	D
B	C	D	E	F	A
A	E	C	B	D	F
F	D	E	C	A	B
D	A	B	F	E	C
C	F	A	D	B	E

Treatment	A	B	C	D	E	F
Extra nitrogen	0	0	0	1	1	1
Extra phosphate	0	1	2	0	1	2

Ronald A. Fisher and Frank Yates



Block designs (1)

Steiner triple system:
A system of three-element subsets of $\{1, \ldots, n\}$ such that each pair $i, j \in \{1, \ldots, n\}$ occurs in exactly one subset.

1	2	3	4	5	6	7
2	3	4	5	6	7	1
4	5	6	7	1	2	3

The number of blocks in each Steiner triple system is $n(n-1)/6$ (the number of pairs is $n(n-1)/2$, each block contains three of them).

Steiner triple system exists if and only if $n = 6k + 1$ or $n = 6k + 3$ (Thomas P. Kirkman 1847, Jakob Steiner 1853)

Block designs (2)

Block design:
A system of k-element subsets of $\{1, \ldots, n\}$ such that each pair $i, j \in \{1, \ldots, n\}$ occurs in exactly ℓ subsets.

Example: $n = 10$, $k = 4$, $\ell = 2$

0 1 2 3	0 1 4 5	0 2 4 6	0 3 7 8	0 5 7 9
0 6 8 9	1 2 7 8	1 3 6 9	1 4 7 9	1 5 6 8
2 3 5 9	2 4 8 9	2 5 6 7	3 4 5 8	3 4 6 7

The number of blocks in each block design is $b = \ell\frac{n(n-1)}{k(k-1)}$.

Design of experiments: Testing n objects, a single experiment involves k objects. Instead of testing all possible subsets of size k, we choose only b of them in such a way that each pair of objects is tested ℓ times.

Projective plane of order N = block design with blocks corresponding to lines, $k = N + 1$, $\ell = 1$, $n = b = N^2 + N + 1$.

Block designs (3)

Block designs were introduced by F. Yates (*Incomplete randomised blocks*, Annals of Eugenics, 1936)

R. A. Fisher, F. Yates: *Statistical Tables for Biological Agricultural and Medical Research* – tables of block designs

Open problem: For which values of n, k, ℓ does there exist a corresponding block design?

R. A. Fisher (1940): If $k < n$, a necessary condition for the existence of a block design is that $b \geq n$ (an experiment with n objects requires at least n blocks), i.e., $\ell(n-1) \geq k(k-1)$.

Hall's theorem, set-theoretic version

Let $A_1, \dots, A_n$ be subsets of a set X. A system of distinct representatives is a collection of n distinct elements $x_1, \dots, x_n$ such that $x_i \in A_i$ for all $i \in \{1, \dots, n\}$.

Theorem (Philip Hall, 1935)

Subsets $A_1, \dots, A_n$ of a finite set X have a system of distinct representatives if and only if for each $\ell \in \{1, \dots, n\}$, the union of arbitrary ℓ sets from $A_1, \dots, A_n$ contains at least ℓ elements.

Application (Marshall Hall, 1945): A Latin rectangle can be always extended to a Latin square.

Hall's theorem, graph-theoretic version

Theorem

Let $G = (V, E)$ be a bipartite graph with parts V_1 and V_2. There exists a matching covering V_1 if and only if each set of ℓ vertices from V_1 has at least ℓ neighbors.

"Marriage version" (due to H. Weyl, 1949):

Given n distinct objects $a_1, \dots, a_n$ (boys) and n distinct objects $b_1, \dots, b_n$ (girls); moreover a scheme of linkage Q_n according to which an a_i and a b_k are either linked (friends) or not linked. What is the necessary and sufficient condition that the boys can be paired with the girls in such a fashion that in each of the n pairs the partners are friends?

Hall's theorem – a brief history

- D. Kőnig (1916): Each regular bipartite graph has a perfect matching.
- D. Kőnig (1916), G. Frobenius (1917) – matrix version of Hall's theorem: *Let A be an $n \times n$ matrix. Then all terms in the definition of* $\det A$ *are zero, if and only if A contains a zero submatrix with dimensions $k \times \ell$, where $k + \ell = n + 1$.*
- D. Kőnig (1931), J. Egerváry (1931): *The maximum size of a matching in a bipartite graph is equal to the size of a minimum vertex cover.* (vertex cover = collection of vertices incident with each edge of the graph)
- P. Hall (1935): Set-theoretic version, inspired by a result in group theory by van der Waerden
- H. Weyl (1949): Almost-periodic functions, simpler proof
- P. Halmos, H. Vaughan (1950): Short elegant proof of Hall's theorem, extension to infinite systems of sets, harem version of Hall's theorem

Sperner's theorem

Emanuel Sperner, *Ein Satz über Untermengen einer endlichen Menge*, 1928:

Consider an n-element set X. What is the largest possible family of subsets of X such that no subset is contained in another subset?

Observation: The family of all k-element subsets of X has size $\binom{n}{k}$, no k-element subset contains another one.

Theorem

The largest possible family of subsets of X none of which contain any other sets in the family has $\binom{n}{\lfloor n/2 \rfloor}$ elements.

Possible motivation: Given a square-free integer N, what is the maximum number of its positive divisors, no one of which divides any other? (Note: divisor $\leftrightarrow$ set of all primes occuring in its factorization)

Schur's theorem

Theorem

For each $k \in \mathbb{N}$, there exists an $S(k) \in \mathbb{N}$ such that in each partition of the set $\{1, \ldots, S(k)\}$ into k classes, there exist numbers x, y, z belonging to the same class and satisfying $x + y = z$.

partition into k classes = coloring using k colors, existence of a monochromatic solution to $x + y = z$

Example: S(2)=5 1 2 3 4

Issai Schur, *Über die Kongruenz $x^m + y^m \equiv z^m \pmod{p}$*, 1916:

An elegant proof of a result by Leonard E. Dickson: *For a fixed $m \in \mathbb{N}$, there exists a $p \in \mathbb{N}$ such that the congruence $x^m + y^m \equiv z^m \pmod{p}$ has a solution.*
(relation to Fermat's Last Theorem)

Van der Waerden's theorem

Van der Waerden, *Beweis einer Baudetschen Vermutung*, 1927:

Theorem

For each pair $k, l \in \mathbb{N}$, there exists a number $W(k, l) \in \mathbb{N}$ with the following property: If the set $\{1, \dots, W(k, l)\}$ is partitioned in an arbitrary way into k classes, then one class contains an arithmetic progression of length l.

Example: W(2,3)=9 1 2 3 4 5 6 7 8

- Pierre J. H. Baudet's conjecture corresponds to $k = 2$
- The conjecture was stated not only by P. J. H. Baudet, but independently also by I. Schur
- Result published in *Nieuw Archief voor Wiskunde*, became popular after its appearance in *Three pearls of number theory* (Russian 1947, German 1951, English 1952) by Aleksander Khinchin, who learned about the result from van der Waerden in Göttingen

Ramsey's theorem

Frank P. Ramsey, *On a problem of formal logic*, 1930:

Theorem

For each triple $r, n, k \in \mathbb{N}$, there exists an $M \in \mathbb{N}$ such that the following statement holds for each $m \geq M$: If we partition all r-element subsets of the set $\{1, \ldots, m\}$ into k classes, there exists an n-element subset of $\{1, \ldots, m\}$ whose r-element subsets all belong to the same class.

- $r = 1$: Pigeonhole principle
- $r = 2$: If the edges of the complete graph K_m are colored using k colors, there exists a monochromatic complete subgraph with n vertices.
  (*Among six people either at least three of them are mutual strangers or at least three of them are mutual acquaintances.*)
- Main topic of the paper is satisfiability of logical formulas

Erdős-Szekeres theorem

P. Erdős, G. Szekeres: *A combinatorial problem in geometry*, 1935:

Theorem

For every $n \in \mathbb{N}$, there exists an $\ell \in \mathbb{N}$ such that any set of at least ℓ points in the plane in general position has a subset of n points forming a convex polygon.

- First proof based on Ramsey's theorem
- New proof of Ramsey's theorem including an estimate for ℓ
- Second proof based on the following result: *An arbitrary sequence of $rs + 1$ real numbers contains a nondecreasing subsequence of length $r + 1$ or a nonincreasing subsequence of length $s + 1$.*

Four-color problem (1)

An important source of progress in graph theory was the ongoing work on the resolution of the four color conjecture proposed in 1852 by Francis Guthrie.

Although the 1879 proof by Kempe turned out to be incorrect, his idea of reducible configurations and unavoidable sets of configurations finally led to the computer-assisted proof of the conjecture in 1976 by Appel and Haken.

In the meanwhile, some important results were obtained by Philip Franklin (1898–1965). In 1922, he discovered new reducible configurations, and showed that every irreducible map must contain more than 25 regions. Thus, he verified the four color conjecture for each map containing at most 25 regions. The problem of coloring maps makes sense not only in the plane, but on other surfaces as well. In 1934, Franklin improved a result by Percy John Heawood by showing that for the Klein bottle, six colors always suffice.

Four-color problem (2)

George D. Birkhoff (1912): $P(\lambda)$ = the number of ways of coloring the map in λ colors. P is a polynomial of degree n, where n is the number of regions – the chromatic polynomial of a given map. Birkhoff found a formula for the coefficients of P. The four-color problem is equivalent to $P(4) > 0$ for all maps.

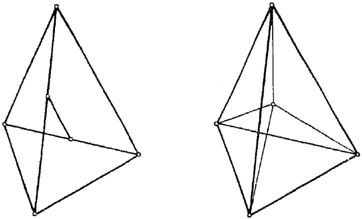
Birkhoff was intrigued by the four-colour problem, and in later years he regretted that he had wasted so much time on it. But he also declared that every great mathematician had at some time attacked the problem, and had, for a while, believed himself successful. From his son, Garrett, we learn that he would ask his wife to prepare suitably complicated maps for him to colour: Mrs. Birkhoff's opinion of this task has not been recorded. (Biggs, Lloyd, Wilson)

Hassler Whitney (1932) – dissertation on graph coloring supervised by Birkhoff; extension of chromatic polynomial to all graphs, a simple proof of formulas for the coefficients; it suffices to study the four-color problem for Hamiltonian graphs

Graph theory – additional results (1)

Characterization of planar graphs:

- Kazimierz Kuratowski (*Sur le problème des courbes gauches en Topologie*, 1930): a graph is planar if and only if it does not contain a subgraph homeomorphic to K_5 or $K_{3,3}$; more general formulation in terms of topological notions (continuum)



- Hassler Whitney (1932, 1933): definition of an abstract dual graph; a graph is planar if and only if it has an abstract dual

Graph theory – additional results (2)

Menger's theorem:

- Karl Menger (*Zur allgemeinen Kurventheorie*, 1927): Let A, B be two disjoint sets of vertices. Then there exist n vertex-disjoint paths between A and B if and only if A, B cannot be separated by deleting n vertices.
- Menger's comment (1981): *Some graph theorists may be surprised to learn that this graph theoretical assertion first came up in 1926 as a lemma in proving an extremely general theorem of set theoretical curve theory.*

Cayley's formula:

- Arthur Cayley (1889): The number of trees on n labeled vertices is n^{n-2}; rigorous proof is missing.
- Heinz Prüfer (*Neuer Beweis eines Satzes über Permutationen*, 1918): Bijection between trees and sequences of numbers from $\{1, \ldots, n\}$ of length $n-2$. (*In how many ways is it possible to connect n cities using a railroad network?*)

Minimum spanning tree (1)

Given a connected weighted graph, find its minimum spanning tree.

The first algorithm for solving this problem was proposed by the Czech mathematician Otakar Borůvka in 1926.

The problem was suggested to Borůvka during World War I by his friend Jindřich Saxel, who worked for the West-Moravian Powerplants, and stated the problem in terms of cities and the distances between them. Borůvka was offered a job with West-Moravian Powerplants, but he declined.

Borůvka's 1926 paper was written in Czech with a German summary, and the whole problem was stated in the language of matrices, whose elements correspond to edge weights. The final paragraph contains the following geometric interpretation:

Consider n points in the r-dimensional space, whose distances are pairwise distinct. The problem is to connect them by a network whose total length is as small as possible.

Minimum spanning tree (2)

Borůvka's description of the algorithm for finding the optimal solution is long and complicated, but becomes much more transparent when reformulated in graph-theoretical language:

Begin by joining each vertex with its nearest neighbor, obtaining a certain forest. For each component, add the shortest edge joining it to a different component, and repeat this step until obtaining a connected graph.

Essentially the same explanation, but without using graph-theoretical terminology, was given in Borůvka's subsequent two-page paper published in a Czech journal aimed at electrical engineers. Borůvka noted that the problem is of importance in the design of electrical networks, and explained the algorithm by means of an example with 40 vertices.

Borůvka's algorithm turns out to be rather efficient, and is the basis of even faster algorithms.

Minimum spanning tree (3)

An alternative algorithm for solving the same problem was proposed by the Czech mathematician Vojtěch Jarník in 1930.

The problem was again formulated in an algebraic way, only the end of the paper provides an intuitive interpretation: *There are n balls connected by* $\frac{1}{2}n(n-1)$ *rods of pairwise distinct weights. The goal is to remove some of these rods in such a way that the balls still hold together, and the mass of the remaining rods is as small as possible.*

Jarník's algorithm in graph-theoretical language: Begin with an arbitrary vertex, and find the shortest edge incident with this vertex, giving rise to a tree with two vertices. Add the shortest edge joining the tree to a vertex that is not included in the tree. Repeat this step until obtaining a tree containing all vertices.

The algorithm is often called Prim's algorithm after Robert Clay Prim, who discovered it independently in 1957.

Major universities in Czechia in the interwar period

Prague:

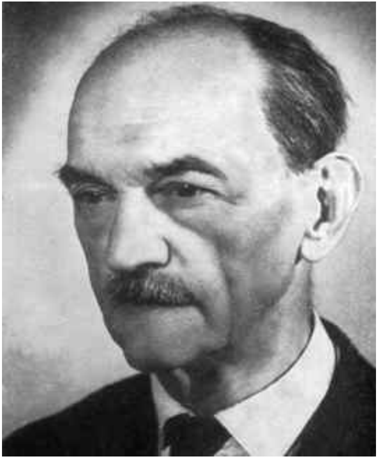
- Czech University in Prague (before 1920) / Charles University (after 1920)
- German University in Prague
- Czech Technical University in Prague
- German Technical University in Prague

Brno:

- Masaryk University (founded 1919)
- Czech Technical University in Brno
- German Technical University in Brno

Czech universities were closed after the German occupation in 1939, and reopened only in 1945. German universities became subordinated to the German Ministry of Education, and were abolished in 1945.

Otakar Borůvka (1899–1995) and Vojtěch Jarník (1897–1970)



Dénes Kőnig – pioneer of graph theory

THEORIE DER ENDLICHEN
UND UNENDLICHEN GRAPHEN

KOMBINATORISCHE TOPOLOGIE DER STRECKENKOMPLEXE

VON

DÉNES KÖNIG

A. O. PROFESSOR AN DER KGL. UNG. JOSEFS-UNIVERSITÄT FÜR TECHNISCHE UND WIRTSCHAFTSWISSENSCHAFTEN IN BUDAPEST

MIT 107 FIGUREN

LEIPZIG 1936
AKADEMISCHE VERLAGSGESELLSCHAFT M. B. H.

Author of first textbook in graph theory: *Theorie der endlichen und unendlichen Graphen* (1936, republished 1950, 1986); English translation *Theory of Finite and Infinite Graphs* (1990)

The status of graph theory in the 1930s (1)

Paul Erdős in 1977:

. . . I cannot help feeling sorry that Dénes Kőnig did not live to see the present flowering of graph theory to which he contributed so much. I myself got interested in graph theory when I was in high school and saw a paper by Kőnig published in the mathematical magazine for high school students. . . .

It is curious how little graph theory and combinatorial analysis was appreciated in those early dark ages. A friend of my parents, a statistician, once said about Dénes Kőnig: "He is great in his art but his art is so small." Ten years later J. H. C. Whitehead, the great English topologist, said about a graph theorist: "He works in the slums of topology." When I first got to Princeton in 1938, I was surprised how many of the topologists looked down upon the four colour problem and considered it an unimportant side issue.

The status of graph theory in the 1930s (2)

William Thomas Tutte in the introduction to the English translation of Kőnig's textbook:

But the honour of presenting Graph Theory to the mathematical world as a subject in its own right, with its own textbook, belongs to Dénes Kőnig. Low was the prestige of Graph Theory in the Dirty Thirties. . . . It was the so-called science of trivial and amusing problems for children, problems about drawing a geometrical figure in a single sweep of the pencil, problems about threading mazes, and problems about colouring maps and cubes in cute and crazy ways. It was too hastily assumed that the mathematics of amusing problems must be trivial, and that if noticed at all it need not be rigorously established. Students tempted by Graph Theory would be advised by their supervisors to turn to something respectable or even useful, like differential equations. I am reminded that my own most recent research in Graph Theory has involved differential equations. Mathematics is One, after all.

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